

A first DSGE model: a productivity shock in a real business cycle economy

The ideas in six lines

1. A **DSGE** model is a small complete economy: people optimise, markets clear, shocks arrive at random.
2. The **steady state** is where the model rests when no shocks arrive.
3. The **Euler equation**: giving up one unit today must pay back, in expected discounted terms, tomorrow.
4. **Log-linearising** around the steady state gives linear rules that can be solved.
5. An **impulse response** traces each variable after one shock, in % deviations from the steady state.
6. **Consumption smoothing**: a short-lived gain is mostly invested; a long-lived one is consumed more.

The formulas

$$Y_t = Z_t K_t^\alpha, \quad K_{t+1} = (1 - \delta)K_t + Y_t - C_t, \quad \ln Z_{t+1} = \rho \ln Z_t + \varepsilon_{t+1}$$

$$\frac{1}{C_t} = \beta \mathbb{E}_t \left[\frac{1}{C_{t+1}} (\alpha Z_{t+1} K_{t+1}^{\alpha-1} + 1 - \delta) \right]$$

Steady state: $\frac{K}{Y} = \frac{\alpha}{1/\beta - 1 + \delta}.$

Worked example

Quarterly model: $\beta = 0.99$, $\alpha = 0.33$, $\delta = 0.025$.

- $K/Y = 0.33/(1/0.99 - 1 + 0.025) = 0.33/0.0351 \approx 9.4$ quarters of output (about 2.35 years).
- A 1% productivity shock with $\rho = 0.9$ (fixed labour, log utility): on impact output +1%, investment about +3.5%, consumption about +0.2%.
- With $\rho = 0.99$: consumption about +0.5%, investment about +2.6%.

Four traps

- “Output on impact depends on persistence.” No. With capital and labour fixed this quarter, output moves one for one with productivity.
- “A more persistent shock is invested more.” No. It is consumed more.
- “Log-linearised results are exact.” No. They approximate the model near the steady state.
- “The steady state means nothing moves.” It means the economy rests there without shocks.

Words to know

steady state · Euler equation · log-linearisation · impulse response · persistence · consumption smoothing · representative household

Try these

1. $\beta = 0.98$, $\alpha = 0.3$, $\delta = 0.02$. What is the steady-state K/Y ? (Two decimals.)
2. Productivity jumps by 2%. Capital and labour are fixed this quarter. By how much does output rise on impact?
3. The shock becomes more persistent. Does the impact response of consumption rise or fall?
4. $\rho = 0.9$. What share of a productivity shock is left after 4 quarters?
5. In words: what does the Euler equation set equal?
6. Why does investment rise much more than consumption after a temporary shock?

Answers

1. $0.3/(1/0.98 - 1 + 0.02) = 0.3/0.0404 \approx 7.42$.
2. 2%, from $Y = ZK^\alpha$ with K fixed (and labour fixed).
3. It rises: the household is richer for longer, so it consumes more now.
4. $0.9^4 \approx 0.656$: about 66%.
5. The marginal utility of consuming today, and the discounted expected marginal utility of consuming tomorrow times the return on saving.
6. Households smooth consumption. They save most of a short windfall, and in this model saving is investment.