

The Solow model: capital, technology and growth

The ideas in six lines

1. Output per worker depends on capital per worker: $y = k^\alpha$, with $0 < \alpha < 1$.
2. **Diminishing returns**: each extra unit of capital adds less output than the one before.
3. Capital per worker stops changing where saving just covers depreciation and new workers: the **steady state**.
4. A higher saving rate raises the **level** of output per worker, not its long-run **growth rate**.
5. Only continuing **technological progress** keeps output per worker growing, at the rate of that progress.
6. Steady-state consumption is highest at the **golden rule**: $s = \alpha$ when $y = k^\alpha$.

The formulas

$$k_{t+1} = \frac{s k_t^\alpha + (1 - \delta) k_t}{1 + n}$$

$$s k^{*\alpha} = (n + \delta) k^* \Rightarrow k^* = \left(\frac{s}{n + \delta} \right)^{\frac{1}{1-\alpha}}, \quad y^* = k^{*\alpha}$$

Steady-state consumption per worker: $c^* = (1 - s) y^*$.

Worked example

$s = 0.3, n = 0.01, \delta = 0.05, \alpha = 0.33$:

- $k^* = (0.3/0.06)^{1/0.67} = 5^{1.49} \approx 11.05$
- $y^* = 11.05^{0.33} \approx 2.21$
- $c^* = 0.7 \times 2.21 \approx 1.55$

Raising s from 0.20 to 0.30 (model of the chapter): output per worker ends up about 22% higher. Consumption per worker falls first, from 80 to about 71 (output in year 0 = 100), and is back above 80 after about 25 years.

Four traps

- **"More investment means faster growth forever."** No. It raises the level; growth fades.
- **"More saving is always better."** No. Past the golden rule, consumption falls.
- **"AI spending is technological progress."** Servers and data centres are capital. Progress means producing more with the same inputs.
- **"Growth fading means the policy failed."** No. The level stays higher.

Words to know

capital per worker · diminishing returns · depreciation · steady state · level effect · growth effect · golden rule · technological progress

Try these

1. $s = 0.2, n = 0.02, \delta = 0.06, \alpha = 0.5$. What is k^* ?
2. With the numbers in question 1, what is y^* ?
3. Without technological progress, the saving rate rises. What happens to the long-run growth rate of output per worker?
4. $y = k^{0.4}$. What is the golden-rule saving rate?
5. Technology grows by 1.5% a year. What is the long-run growth rate of output per worker?
6. Why does consumption per worker fall at first when the saving rate rises?

Answers

1. $k^* = (0.2/0.08)^{1/0.5} = 2.5^2 = 6.25$.
2. $y^* = 6.25^{0.5} = 2.5$.
3. It goes back to zero. Growth is faster only during the transition; the level ends higher.
4. $s = \alpha = 0.4$.
5. 1.5% a year.
6. A larger share of the same output is saved at once, while the extra capital takes years to build up.